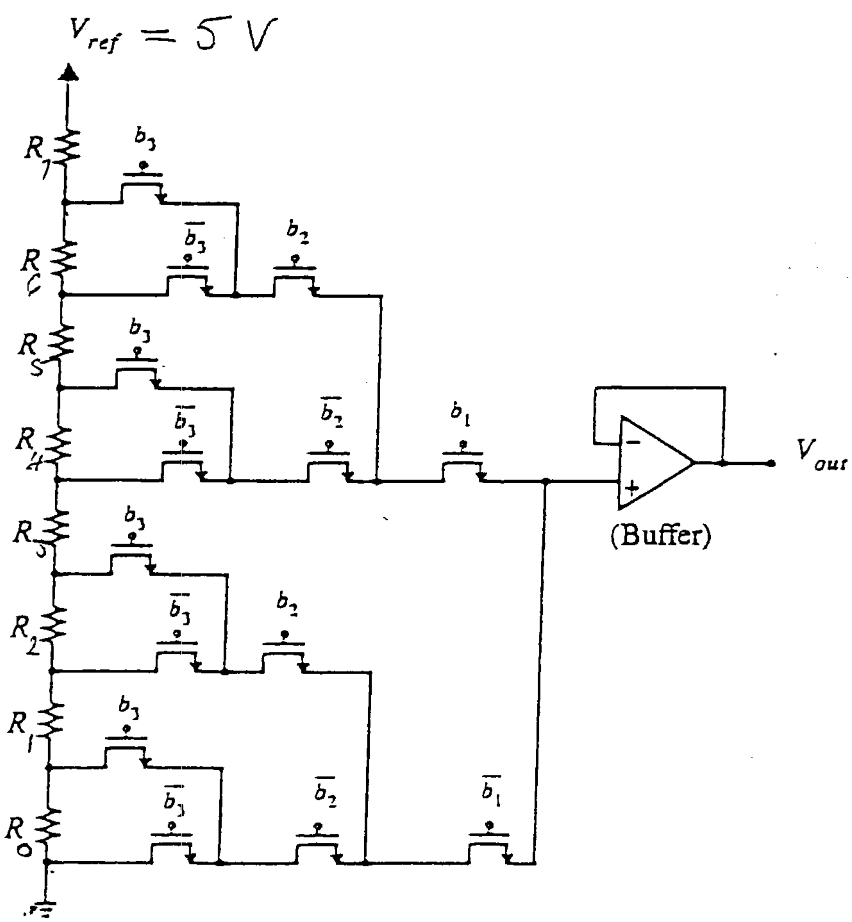
1. In the 3-bit DAC shown, process variations cause the <u>k</u> th resistor to have an error $r_k = 0.02kR_o$ for k [redacted] 1, 2, ..., 7.

a. What will be the output voltage levels?

b. What will be the DNL, in volts and LSBs?

c. What will be the end-point INL, in volts and LSBs?



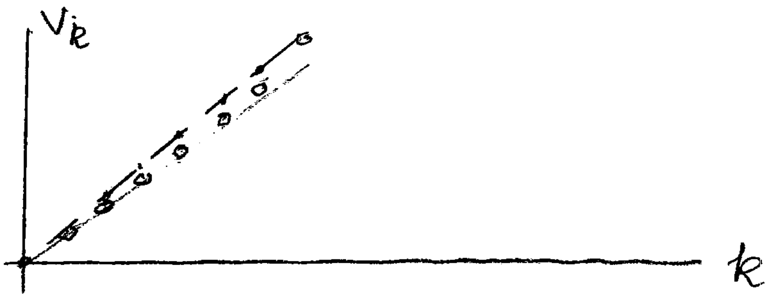
1. a. $R_i = R_o(1 + 0.02 i)$

$$V_k = V_{ref} \frac{\sum_{i=0}^{k-1} R_o(1+0.02i)}{\sum_{i=0}^{7} R_o(1+0.02i)} = \frac{V_{ref}}{8.56} k\left(0.99 + \frac{k}{100}\right)$$

$$\sum_{i=0}^{n} i = 0 + 1 + \dots + (n-1) + n = \frac{n(n+1)}{2}$$

$V_0 = 0\ V$, $V_1 \cong 0.584\ V$, $V_2 \cong 1.18\ V$, ...

b. The largest step occurs between V_6 & V_7: $\Delta V_{67} \cong 0.654\ V$.



$V_{LSB} = V_{ref} 2^{-3} = 0.625\ V$

$DNL_{max} = 0.654 - 0.625 = 0.029\ V \rightarrow 0.046\ LSB$

c. $INL = |V_k - k V_7/7| =$

$$= \frac{V_{ref}}{8.56} k\left[0.99 + \frac{7}{100} - \left(0.99 + \frac{k}{100}\right)\right] =$$

$$= \frac{V_{ref}}{856} k(7-k). \quad \frac{d}{dk} INL \Rightarrow 0 \rightarrow k = 3.5$$

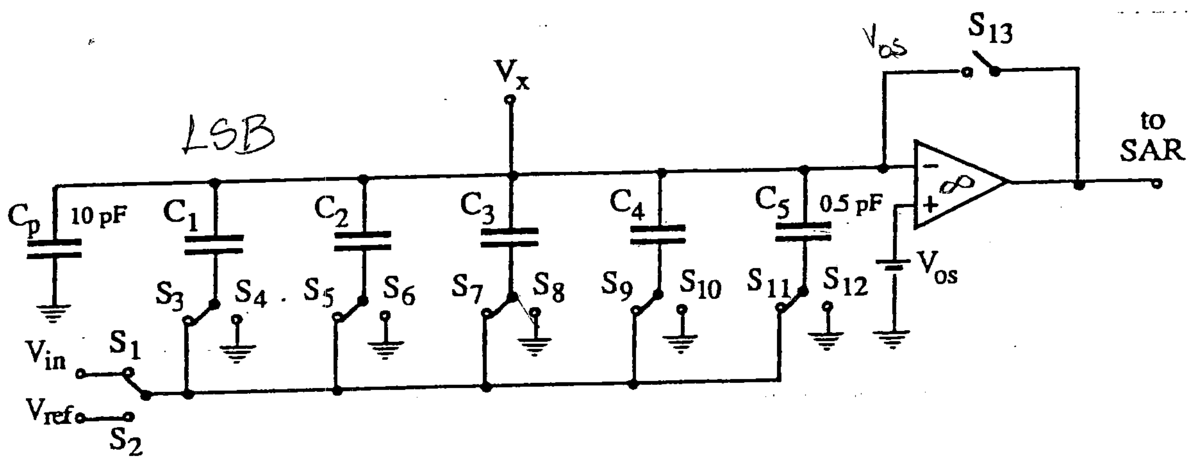
Max INL for V_3 & V_4: $\frac{5}{856} 12 = 70\ mV = 0.112\ LSB$

1 (60%) For the 4-bit A/D converter shown, $V_{ref} = 2.5$ V and $V_{in} = 1.76$ V. The smallest capacitor in the circuit is 0.5 pF, and the parasitic capacitance C_p is 10 pF. The offset voltage of the op-amp is $V_{OS} = 4$ mV.

(a) Give all capacitor sizes.

(b) For each conversion cycle, give the value of V_X and the resulting bit.

(c) What is the difference between V_{in} and the analog equivalent of the digital output, in volts and in LSB's ?



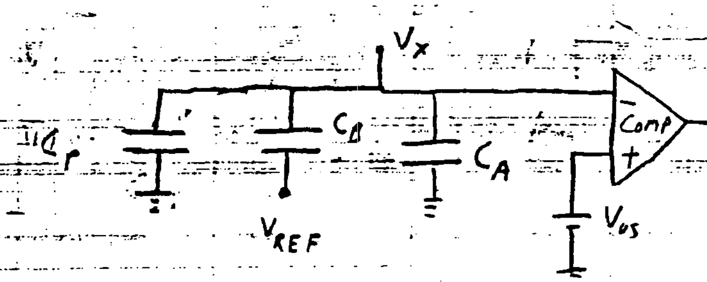
a) $C_1 = 4.0\,pF$

$C_2 = 2.0\,pF$

$C_3 = 1.0\,pF$

$C_4 = 0.5\,pF$ ✓

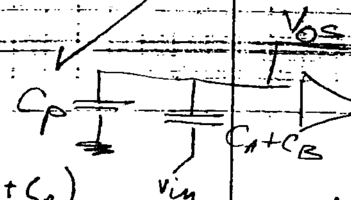
b) Let's model the circuit as



$q_A = C_A V_x$

$q_B = C_B (V_x - V_{REF})$

$q_p = C_p V_x$



Nice!

But $q_A + q_B + q_p = V_{os} C_p - (V_{in} - V_{os})(C_A + C_B)$

So

$$C_A V_x + C_B (V_x - V_{REF}) + C_p V_x = V_{os} C_p - (V_{in} - V_{os})(C_A + C_B)$$

$$V_x (C_A + C_B + C_p) = V_{REF} C_B + V_{os} C_p + (V_{os} - V_{in})(C_A + C_B)$$

$$V_x = \frac{V_{REF} C_B + V_{os} C_p + (V_{os} - V_{in})(C_A + C_B)}{C_A + C_B + C_p} \quad \text{(8)} \quad = 8 + 10$$

Step 1: $C_A = 4pF$ (C_1) $C_B = 4pF$

(Using Calculator to evaluate)

$V_x = -.22\ V$ $b_3 = 1$ ← ✓

$-0.22267V$

Step 2: $C_A = 2pF$ $C_B = 6pF$

$V_x = +0.0551$ $b_2 = 0$ ← ✓

Step 3: $C_A = 3pF$ $C_B = 5pF$

$V_x = -0.094$ $b_1 = 1$ ← ✓

Step 4: $C_A = 2.5pF$ $C_B = 5.5pF$

$V_x = -0.014$ $b_0 = 1$ ← ✓

c)

1011 binary 11 decimal

quant. error

$$\left| \left(\frac{11}{16} \cdot V_{REF} \right) - V_{in} \right| = 0.04125\ V$$

or $\frac{0.04125\ V}{\frac{1}{16} \cdot 2.5V}$ LSB's $= 0.264$ LSB ✓